Model Theory: Philosophy, Mathematics, and Language

Munich Center for Mathematical Philosophy, LMU Munich

Abstracts Booklet

Program

Day 1 - Monday, January 9, 2017

10:00-10:15	Welcome
10:15-11:30	Gila Sher (USCD): The Foundational Role of Model Theory
11:30-11:45	Coffee break
11:45-13:00	Tim Button (Cambridge): Internal Categoricity Results and Internalism in the Philosophy of Mathematics
13:00-14:30	Lunch break
14:30-15:15	Neil Barton (Kurt Gödel Research Center, University of Vienna): Mathematics as the Science of (Different Kinds of) Structures
15:15-15:30	Coffee break
15:30-16:15	Andrei Rodin (Russian Academy of Sciences and Saint-Petersburg State University): Categorical Model Theory and the Semantic View of Theories
16:15-16:20	—-
16:20-17:05	Maciej Kłeczek (Bielefeld): The Meaning of a First-Order Formula, Compositionality and Alphabetic Innocence
17:05-17:20	Coffee break
17:20-18:35	Dag Westerståhl (Stockholm): Models, Formal Languages, and Truth in Natural Language Semantics

Day 2 - Tuesday, January 10, 2017

10:15-11:30	John T. Baldwin (University of Illinois at Chicago): Philosophical Implications of the Paradigm Shift in Model Theory
11:30-11:45	Coffee break
11:45-13:00	Richard Kaye (Birmingham): Satisfaction Classes for Stratified Logic
13:00-14:30	Lunch break
14:30-15:15	Michał Godziszewski (Warsaw): Short Elementary Cuts in Countable Models of Compositional Arithmetical Truth
15:15-15:30	Coffee break
15:30-16:15	Dimitris Tsementzis (Rutgers): Model Theory in the Univalent Foundations and its Philosophical Prospects
16:15-16:20	—-
16:20-17:05	Alexander Jones (Bristol): Minimal Adequacy and Semantic Truth
17:05-17:20	Coffee break
17:20-18:35	Georg Schiemer (Vienna / LMU): Geometrical Roots of Model Theory
19:00	Conference dinner

Day 3 - Wednesday, January 11, 2017

10:15-11:30	Yoad Winter (Utrecht): Partial Models and the Symmetry-Collectivity Hypothesis
11:30-11:45	Coffee break
11:45-13:00	Hartry Field (NYU): Generalizing Fuzzy Logic and its Model theory, for Semantic Paradoxes (and Vagueness)
13:00-13:15	Coffee Break
13:15-14:00	Juliette Kennedy (Helsinki): Tarski and "The Mathematical"
From 14:00	Lunch and social activities

Day 4 - Thursday, January 12, 2017

10:15-11:30	Menachem Magidor (Hebrew University): Is Independence Relevant?
11:30-11:45	Coffee break
11:45-13:00	Thomas Ede Zimmermann (Frankfurt): Models and Worlds in Linguistic Semantics
13:00-14:30	Lunch break
14:30-15:15	Fabrizio Calzavarini (Turin): The Cognitive Reality of Model-Theoretic Semantics for Natural Language
15:15-15:30	Coffee break
15:30-16:15	Bernhard Nickel (Harvard): Generics and Conservativity
16:15-16:20	—-
16:20-17:05	Ali Abasnezhad (LMU): Why Tolerance May Not Be Preserved In Model Theoretic Frameworks
17:05-17:20	Coffee break
17:20-18:35	Volker Halbach (Oxford): Axiomatic Semantics and the Substitutional Theory of Logical Consequence

Abstracts

Gila Sher (USCD): The Foundational Role of Model Theory

The talk investigates model theory's foundational roles in philosophy – both the roles it has and the roles it does not have. My focus is on its potential roles in a positive foundation for logic and in criticisms of realism and the correspondence theory of truth. Among the critical questions I raise are: (i) Can model theory be used as a framework for understanding theories in general or just logical theories? Does model theory have significant ramifications for truth in general or just for logical truth? (ii) What do models represent (if anything) and how? (iii) What is the relation between the philosophical significance of model theory and its mathematics? Is its foundational role limited by features (peculiarities, weaknesses) of the mathematical theory in which it is embedded? If not, how does it transcend such limitations?

Tim Button (Cambridge): Internal Categoricity Results and Internalism in the Philosophy of Mathematics

Elementary results in first-order model theory (Compactness, Löwenheim–Skolem) raise serious problems for any position which combines a (moderate) naturalism with the idea that mathematical structure is well explicated in terms of isomorphism types. That is the gist of Putnam's 'Models and Reality' (1980). But, aside from a very brief sketch at the end of that paper, it is not clear what positive philosophy of mathematics Putnam wanted to advance. I shall outline how recent work concerning *internal categoricity* results might provide an answer. (The recent work is due to Parsons, McGee, Lavine, Väänänen & Wang, and work I have coauthored with Walsh.) They suggest that the (moderate) naturalist might adopt an *internalist* position, according to which: *the use of a second-order deductive theory (of arithmetic or set theory) settles* ***that*** *every sentence in the language of that theory is either determinately true or determinately false, even though the truth values of certain sentences cannot be decided within that theory.*

Neil Barton (Kurt Gödel Research Center, University of Vienna): Mathematics as the Science of (Different Kinds of) Structures

Structuralism in mathematics (the view that the subject matter of mathematics consists of structures rather than objects) has formed a substantial area of philosophical research since the 1960s. Often structuralists distinguish between *particular* structures (intended categorical or canonical) and *general* structures (explicitly intended non-categorical). This paper informs this distinction by bringing to bear philosophical considerations and results from contemporary model theory on the issues. We argue that holding this distinction to be exhaustive blurs the philosophical and mathematical landscape, and closes off several plausibly attractive options for the structuralist. Instead, we advocate that the distinction between particular and general should be viewed as a matter of degree, and that there is a third and distinct kind of structure (namely being *particular in a cardinal*).

Andrei Rodin (Russian Academy of Sciences and Saint-Petersburg State University): Categorical Model Theory and the Semantic View of Theories

Categorical Model theory (CMT) stems from the functorial semantics of algebraic theories proposed by Lawvere in his thesis back in 1963. Today this theory uses a family of concepts of model none of which is fairly standard. The technical concepts of model considered in CMT lack so far any generally accepted epistemological underpinning. It remains unclear whether or not the classical Tarskian notion of model based on the T-schema applies in CMT in all cases. I argue that this classical notion is not adequate for accounting for models of Homotopy Type theory. As a remedy I propose a novel concept of model, which agrees with CMT and supports a constructive version of the Semantic View of scientific theories. Finally, I argue that this version of the Semantic View is more adequate to the existing scientific practice than the standard version.

Maciej Kłeczek (Bielefeld): The Meaning of a First-Order Formula, Compositionality and Alphabetic Innocence

We focus on the concept of the meaning of a first-order formula. Initially, we discriminate between global and local approaches towards explicating the notion of the meaning of a first-order formula. The former approach derives from abstract model theory. Whereas the latter identifies the meaning of a first-order formula in a structure with a relation defined over the universe at hand. Primarily, local approaches to the meaning of a first-order formula stem from the algebraization of satisfaction relations. We single out the algebraization of the standard satisfaction relation for ordinary first-order languages relatively to the class of cylindric set algebras of dimension α. We survey the relevant results from this field and discuss two complaints raised in the literature. The first complaint is the so called representationalism. The second charge is the lack of alphabetic innocence. We find the official formulation of representationalism flawed. However, we point that it can be reinstated to a degree in our settings. Next, we direct attention to principle of alphabetic innocence. Admittedly, it is not a well-known semantic principle. Nonetheless, it is endorsed by couple of authors. We provide motivations underpinning alphabetic innocence and define it. Moreover, we argue that it can be interpreted as requiring invariance of meaning assignment under the group of permutation of variables. We state the following precise question " Is it possible to define an alphabetically innocent semantics for ordinary first-order languages ? ". We settle this issue positively by resorting to the well-known concept of definability in a structure.

The construction of our semantics proceeds in few stages. Firstly, we stress that there is no unique way to represent a formula in a finite variable context. We distinguish between orderings of variables parasitic on canonical ordering of variables and ordering suggested by the structure of a formula. We employ the latter ordering and define the operation of bijective simultaneous replacement of free and bound occurrences of variables in a formula. Subsequently, we state the substitution lemma for this operation. Finally, we define Hodges' style semantics for ordinary first-order languages and prove by an appeal to our substitution lemma that this semantics is alphabetically innocent. Our semantics is non-inductively defined and consequently non-compositional.

Finally, we discuss some consequences of our construction. Firstly, we refute the claim that under an alphabetically innocent semantics it is impossible to distinguish within a language a relation and its converse. Secondly, we show that our semantics provides fine-grained criteria of the identity of meaning. There exist logically equivalent formulas (in the standard Tarskian sense) which are non-synonymous.

Dag Westerståhl (Stockholm): Models, Formal Languages, and Truth in Natural Language Semantics

I will present a version of model-theoretic semantics light, trying to isolate what is common to various approaches to formal semantics for natural language that use a model-theoretic framework. The idea is extremely simple, and not very original; yet some current versions of textbook model-theoretic semantics appear to be in conflict with it. I illustrate the idea with examples from Montague style semantics, generalized quantifiers in natural language, and epistemic logic. The aim is not to argue for a particular way of doing model-theoretic semantics, but to make it clearer what one should expect, and not expect, from applying the technology of model theory to natural language semantics.

John T. Baldwin (University of Illinois at Chicago): Philosophical Implications of the Paradigm Shift in Model Theory

The paradigm shift that swept model theory in the 1970's really occurred in two stages. During the first stage in the 1950's and 1960's the focus switched from the study of properties of logics to properties of theories. In the second stage, Shelah's decisive step was to move from merely identifying some fruitful properties (e.g. complete, model complete, $\aleph_1$-categorical) that might hold of a theory to a *systematic classification* of complete first order theories. Model theorists now undertake a systematic search for a finite set of syntactic conditions which divide first order theories into disjoint classes such that models of different theories in the same class have similar mathematical properties. With this framework one can compare different areas of mathematics by checking where theories formalizing them lie in the classification.

From the standpoint of the philosophy of mathematical practice the focus is changed from justifying the *reliability* of mathematical results to the *clear* understanding and organization of mathematical concepts. This shift sheds light on a number of philosophical issues. For example, how does formal logic play a significant role in mathematics beyond the old metaphor of 'the analysis of methods of reasoning'? Why does mathematics focus more on classes of structures than individual structures? What constitutes a paradigm shift in mathematics?

Michał Godziszewski (Warsaw): Short Elementary Cuts in Countable Models of Compositional Arithmetical Truth

Motivated by the project of building model theory for axiomatic systems of compositional arithmetical truth predicate (i.e. models of arithmetic admitting satisfaction classes), we consider a qestion on countable recursively saturated models of Peano Arithmetic, namely: let $\mathcal{M} \models PA$ be a countable recursively saturated model and let $a, b \in M$. Suppose that $(\mathcal{M}, \mathcal{M}(a)) \equiv (\mathcal{M}, \mathcal{M}(b))$. Does it follow that $(\mathcal{M}, \mathcal{M}(a)) \cong (\mathcal{M}, \mathcal{M}(b))$? We report the state of art (the answer is negative for the case in which $\mathcal{M}(a)$ is a coshort elementary cut of $\mathcal{M}$), provide information concerning main methods used in the field and give a partial answer to the main question, i.e. we prove that if $\mathcal{M}(a)$ and $\mathcal{M}(b)$ are short elementary cuts of $\mathcal{M}$ and the family of sets coded in $\mathcal{M}$ (aka the Standard System of $\mathcal{M}$) contains sets definable in the standard model, then the naswer is positive, i.e. then the equivalence between such elementary pairs implies that they are isomorphic. We further prove that if $SSy(M)$ contains a non-arithmetic set, then M has gaps that do not realize complete arithmetic types.

Dimitris Tsementzis (Rutgers): Model Theory in the Univalent Foundations and its Philosophical Prospects

Model theory, as a branch of mathematical logic, has so far been carried out exclusively in a set-theoretic setting. The Univalent Foundations of mathematics (UF) differ from set-theoretic foundations in essential ways. For instance, they take the point of view that spatial notions (e.g. "point" and "path") are fundamental, rather than derived, and that all of mathematics can be encoded in terms of them. Within UF there is thus a new kind of model theory, with a spatial, rather than set-theoretic, semantics. In this talk, I will first introduce the relevant key ideas of UF and UF-based model theory, explaining how it differs from set-theoretic model theory. Then I will outline philosophical and technical prospects for the new model theory.

Alexander Jones (Bristol): Minimal Adequacy and Semantic Truth

In my talk I will look at a new model-theoretic criterion of what it means for a typed (non self-referential) theory of truth to be 'minimally adequate' over arithmetic. This criterion is a natural strengthening of the equivalence schema for nonstandard models with their own (nonstandard) theory of syntax. Ensuring that a theory of truth meets this criterion has a lot of nice proof-theoretic characterisations, but these are nonconservative extensions of arithmetic, which raises a challenge to the deflationist who accepts conservativity as essential to their view. I will argue that this is not so problematic for the deflationist, however, and can even offer some support for the view.

Georg Schiemer (Vienna / LMU): Geometrical Roots of Model Theory

The talk will focus on one of the key contributions to modern structural mathematics, namely Hilberts *Foundations of Geometry* (1899) and its mathematical roots in nineteenth-century projective geometry. A central innovation of Hilberts book was to provide semantically minded independence proofs for various fragments of Euclidean geometry, thereby contributing to the development of the model-theoretic point of view in logical theory. Though it is generally acknowledged that the development of model theory is intimately bound up with innovations in 19th century geometry (in particular, the development of non-Euclidean geometries), so far, little has been said about how exactly model- theoretic concepts grew out of methodological investigations within projective geometry. This talk is supposed to fill this lacuna and investigates this geo- metrical prehistory of modern model theory, eventually leading up to Hilberts *Foundations*. The research presented here is based on joint work with Günther Eder (University of Vienna).

Yoad Winter (Utrecht): Partial Models and the Symmetry-Collectivity Hypothesis

Natural language predicates like *cousin of* and *identical to* are logically symmetric. We start out from the hypothesis that symmetry in natural language reflects an ability to classify collections of entities. Thus, when speakers identify a binary predicate R as symmetric, this is due to a linguistic derivation of R from a unary predicate P_R over sets, where for all entities x and y: $R(x,y) \Leftrightarrow P_R(\{x,y\})$. For example: A is B's cousin if and only if A and B are *cousins*. This account is extended to *partial relations*. We observe the following entailment pattern:

(1) The drunk conversed with the policeman $\Rightarrow$ The policeman *conversed with* the drunk

(2) The drunk conversed with the statue $\not\Rightarrow$ The statue *conversed with* the drunk

To account for this pattern, partial relations like *converse with* are restricted to domains $H \times E$, where $H \subseteq E$. This leads to a generalized formulation of our symmetry-collectivity hypothesis: when R is a binary predicate over $H \times E$, symmetry of R over $H \times H$ is defined using a unary predicate P_R over subsets of H, where for all entities x and y in $H : R(x,y) \Leftrightarrow P_R(\{x,y\})$. For example: for all *humans* A and B, A *conversed with* B if and only if A and B *conversed*.

Desirable implications of this account are: (i) symmetry of predicate constants is encoded in the type system, not in *ad hoc* meaning postulates; (ii) "selectional restrictions" (human, animate, movable etc.) are accounted for; (iii) violations of such restrictions are tolerated; (iv) selectional restrictions are gradable: both arguments of a binary predicate may be required to be human (animate, movable etc.) with one requirement weaker than the other.

Hartry Field (NYU): Generalizing Fuzzy Logic and its Model theory, for Semantic Paradoxes (and Vagueness)

Lukasiewicz continuum-valued logic has been popular in dealing with vagueness, and prominent logicians (e.g. Thoralf Skolem and C. C. Chang) have been very interested in its application to the semantic, property-theoretic and set-theoretic paradoxes. But it isn't ultimately workable for either vagueness or the paradoxes. This talk will sketch how to generalize it to make it work—not for set theory, because of extensionality, but for truth and properties, and also for vagueness. I'll avoid technical details, but give enough of the idea so that those technically inclined shouldn't have much problem filling them in.

Juliette Kennedy (Helsinki): Tarski and "The Mathematical"

Much of Tarski's logical work is characterized by the idea of giving "mathematical" (or "very mathematical") analogues of metamathematical definitions or concepts. Vaught suggests in his account of Tarski's work in model theory [1], that Tarski's guiding principle, as one might call it, resting as it does on the distinction between the "purely mathematical" and the metamathematical, may not have a precise content, "as a precise distinction between "mathematical" and "metamathematical" might well be considered to be impossible because of Tarski's definition of truth."

In this talk I will discuss Tarski's distinction, as well as Vaught's objection to it. I will suggest that Tarski's guiding principle has persisted in the form of a paradigm emphasizing pure semantics in certain research streams in model theory.

I will conclude by arguing that the concept of so-called symbiosis makes the relationship between set-theoretic definability and "the mathematical" precise, contra Vaught. There is a way of viewing the correspondence between the metamathematical and the mathematical which preserves the spirit of Tarski's guiding principle.

This is joint work with Jouko Väänänen.

[1] R. Vaught, "Alfred Tarski's Work in Model Theory", The Journal of Symbolic Logic, Vol. 51, No. 4 (Dec., 1986), pp. 869-882

Menachem Magidor (Hebrew University): Is Independence Relevant?

The phenomena of independence in set theory is a great challenge to the philosopher of mathematics. The fact that we have such a wide spectrum of possible set theories is mind boggling. Independence is not limited to purely set theoretic problems, but appears in many other fields of Mathematics, like Algebra, Topology and Analysis. This raises the spectre of mathematics splitting into mutually incompatible subdisciplines. But is independence really relevant to the practice of Mathematics?

There are several approaches to exorcize the "ugly monster of independence". (in the language of the late Paul Erdős) . Of course we can not hope to eliminate independence completely in view of

Gödel incompleteness theorem. But there were several attempts to minimize its impact on the working mathematician. One approach (e.g. By Sol Feferman) is to claim that "real mathematics" is really done in a small fragment of second order number theory, where there are much less cases of "natural problems" which are shown to be independent.

Another related approach is to claim that the problems which really concerns the working mathematician are not independent and the examples of independent problems in the different mathematical domains are not intrinsically belonging the domain, but are really set theoretical problem in disguise. Another approach is to question the the foundational role of set theory.

A current trend in set theory is to try and extend the accepted axioms to a stronger system in which important classes of independent problems will be decided. This approach faces the difficulty of deciding which axioms should be adapted.

In this talk we shall try to analyse the difficulties that each of these approaches faces. We do not promise an answer to the question in the title.

Thomas Ede Zimmermann (Frankfurt): Models and Worlds in Linguistic Semantics

The model-theoretic tradition in natural language semantics (going back to Montague (1970)) distinguishes three levels of denotations:

a) global meaning, which varies across a space of intensional models (= Model Space);

b) regional content, which varies across a model-dependent space of worlds or indices (= Logical Space);

c) local reference, which is both model- and index-dependent.

Though frequently identified, the levels a) and b) must be sharply distinguished. In particular:

- Variation across Model Space does not reflect semantic features of the object language but rather the semanticist's knowledge about it (Zimmermann 1999).
- While Logical Space(s) need(s) to be as large as possible, Model Space ought to be as small as possible, ideally consisting of [the isomorphism class of] the intended model.

This talk will concentrate on a case study on quantification over alternative intensions (Rooth 1985) that takes a closer look at the relation between a) and b) and confirms the above assessment.

References
Montague, R.: 'Universal Grammar'. *Theoria* **36** (1970), 373-398.
Rooth, M.: *Association with Focus*. University of Massachusetts at Amherst dissertation 1985.
Zimmermann, T. E.: 'Meaning Postulates and the Model-Theoretic Approach to Natural Language Semantics'. *Linguistics and Philosophy* 22 (1999), 529-561.

Fabrizio Calzavarini (Turin): The Cognitive Reality of Model-Theoretic Semantics for Natural Language

In contemporary philosophy of language and linguistics, it is generally assumed that model-theoretic semantics for natural language [MTS-NL] can also provide an explanatory account of meaning as a cognitive phenomenon (e.g., some body of semantic knowledge that speakers possess, or some set of psychological processes underlying human semantic abilities). Such a cognitivist view is the natural extension of the Chomskian approach to syntax, and has been explicitly endorsed in many introductory textbooks for both Montagovian and Davidsonian MTS-NL. In this talk, I will argue that the association of cognitivism and MTS-NL is controversial. The recursive components of MTS-NL could have some empirical import in the study of the compositional aspects of semantic competence. However, MTS-NL is only partially adequate to deal with the lexical aspects of human semantic competence. In philosophy of language, a distinction has been proposed between two aspects of lexical competence (Marconi 1997). The first aspect, i.e. inferential competence, is the ability to deal with the network of semantic relations among lexical units of a natural language, underlying such performances as semantic inference, paraphrase, definition, retrieval of a word from its definition, finding a synonym, and so forth. In MTS-NL, this aspect of lexical competence can be easily modelled by means of Carnapian meaning postulates, i.e. universally quantified (bi)conditional statements that constrain the extensions of the constant that appear in them. The second aspect of lexical competence, i.e. referential competence, concerns the mapping between words and objects, events and circumstances in the world. Critically, the referential aspect of lexical competence is problematic for MTS-NL: no amount of meaning postulates is going to explicate this ability. It is possible to think that the problem could be simply solved by integrating MTS-NL with an adequate account of an ordinary speaker?s referential competence. However, it seems that a deep philosophical problem affects any attempt to provide an integration of this sort for MTS-NL.

Bernhard Nickel (Harvard): Generics and Conservativity

Generic sentences, such as “ravens are black” or “tigers have stripes” seem to express a generalization of some sort. This suggests the hypothesis that generics contain an unpronounced quantificational element at LF, gen. However, this hypothesis faces several problems, too, including the apparent failure of conservativity. For example, ”ravens are black” does not seem to be equivalent to “ravens are ravens that are black”. I argue that the apparent failure of conservativity is merely apparent, and that sentences like “ravens are ravens that are black” or “ravens are black ravens” are unacceptable for independent reasons of uninterpretability. The same analysis also explains the status of “prime numbers are odd”, which is a challenging counterexample for many theories of generics, but which receives a natural treatment on the present account.

Ali Abasnezhad (LMU): Why Tolerance May Not Be Preserved In Model Theoretic Frameworks

Tolerance principle, i.e. insensitivity of the justice with which a vague predicate is applied for sufficiently small changes, is highly intuitive but also susceptible to sorites paradox. While most theories of vagueness reject Tolerance in order to avoid the paradox, recently some philosophers, including Cobreros et al. (2012) and Zardini (2008), have attempted to semantically preserve Tolerance without facing the paradox. In this presentation, however, I argue that Tolerance principle may not be preserved within a standard model-theoretic framework. That is, both Cobreros et al. (2012) and Zardini (2008) fail to truly maintain Tolerance. In the end, I show that Tolerance can be preserved in a game semantic framework.

Volker Halbach (Oxford): Axiomatic Semantics and the Substitutional Theory of Logical Consequence

The model-theoretic definition of logical consequence suffers from various deficiencies, as Etchemendy and others have argued. For instance, on the model-theroetic account it is far from clear that logical consequence is truth preserving. These problems arise because semantic notions are reduced to set-theory on the model-theoretic account. I present a substitutional theory of consequence that is based on a primitive axiomatized notion of satisfaction. I argue that this notion comes close to Kreisel 'informal' notion of logical consequence and overcomes the weaknesses of the model-theoretic definition of logical consequence.

Last updated: January 6, 2017